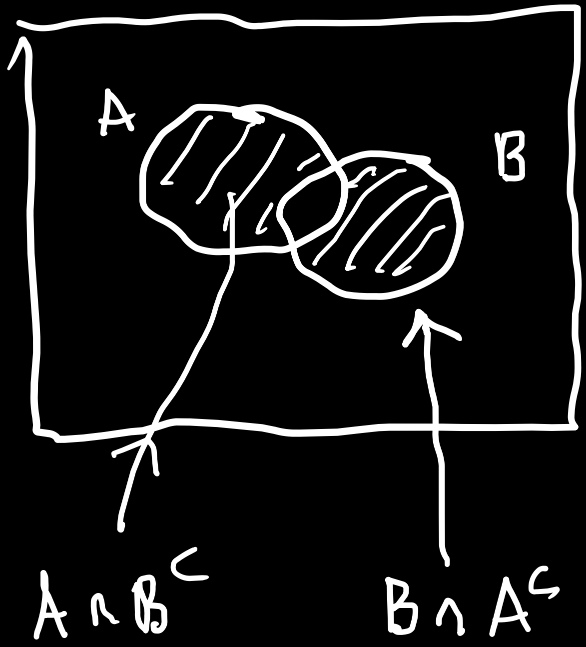
Różnica symetryczna: $A \triangle B = (A \setminus B) \cup (B \setminus A)$




$$A \triangle B = (A \cap B^c) \cup (A^c \cap B)$$

- $x \in A \triangle B \equiv (x \in A) \oplus (x \in B)$

XOR

FAKT:
- $A \triangle \emptyset = A$ $\qquad p \oplus \bot \equiv p$
- $A \triangle A = \emptyset$ $\qquad p \oplus p \equiv \bot$

$p \oplus (q \oplus r) \equiv (p \oplus q) \oplus r$ →

- $A \triangle B = B \triangle A$
- $A \triangle (B \triangle C) = (A \triangle B) \triangle C$

Zbiór potęgowy: $X \in P(A) \longleftrightarrow X \subseteq A$

[ $P(A) =$ "zbiór wszystkich podzb. zbioru $A$" ]

- $P(\emptyset) = ?$ $\qquad X \in P(\emptyset) \longleftrightarrow X \subseteq \emptyset \longleftrightarrow X = \emptyset$

$$P(\emptyset) = \{\emptyset\}$$

$x \subseteq \{a\}$

$\Updownarrow$

$X = \emptyset \vee X = \{a\}$

- $P(P(\emptyset)) \ni X \equiv X \subseteq \{\emptyset\}$

$$\equiv X = \emptyset \vee X = \{\emptyset\}$$

$$P(P(\emptyset)) = \{\emptyset, \{\emptyset\}\}$$

Uwaga: $\begin{cases} \bullet\ \emptyset \in P(X) & (\text{bo } \emptyset \subseteq X \text{ dla dowolnego } X) \\ \bullet\ X \in P(X) & : \text{bo } X \subseteq X. \end{cases}$

- $P(P(P(\emptyset))) = P(\{\emptyset, \{\emptyset\}\}) = (*)$

$$P(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

$$(*) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$$

(P) $x \in (\{a,b\} \cup \{c,d\}) \equiv x \in \{a,b\} \vee x \in \{c,d\}$

$\equiv (x=a) \vee (x=b) \vee (x=c) \vee (x=d)$

$\equiv x \in \{a,b,c,d\}$

<u>Def</u>. $x \in \{a_1, a_2, \dots, a_n\} \overset{\text{def.}}{\equiv} \bigvee_{i=1}^{n} (x = a_i)$

DEF (Para uporządkowana)

$$\boxed{(x,y) \overset{\text{def}}{=} \{\{x\},\{x,y\}\}}$$

[K. Kuratowski]

$(x,y) = \{\{x\},\{x,y\}\}$

TW. $\boxed{(a,b) = (c,d) \equiv (a=c \wedge b=d)}$

D-d ($\Rightarrow$) Załóżmy, że $(a,b) = (c,d)$, czyli

$$\{\{a\},\{a,b\}\} = \{\{c\},\{c,d\}\}.$$

(P1) Załóżmy, że $c=d$. Wtedy

$$\{\{a\},\{a,b\}\} = \{\{c\}\}.$$

$\{a\} \in \{\{c\}\}$; $\{a\} = \{c\}$; $a = c$

$\{a,b\} \in \{\{c\}\}$; $\{a,b\} = \{c\}$; $b \in \{c\}$;

$b = c$

$(c,c) = (a,b)$ $\boxtimes$

$\uparrow c \quad \uparrow c$

P2. $c \neq d$ $\qquad \{\{a\},\{a,b\}\} = \{\{c\},\{c,d\}\}$

- $\{a\} \in \{\{c\},\{c,d\}\}$

$? \; \{a\} = \{c,d\}$

$\left.\begin{array}{l} c = a \\ d = a \end{array}\right] \rightarrow c = d$ sprzeczność

mamy $\{a\} = \{c\}$ czyli $a = c$.

CZYLI

$$\{\{c\},\{c,b\}\} = \{\{c\},\{c,d\}\}$$

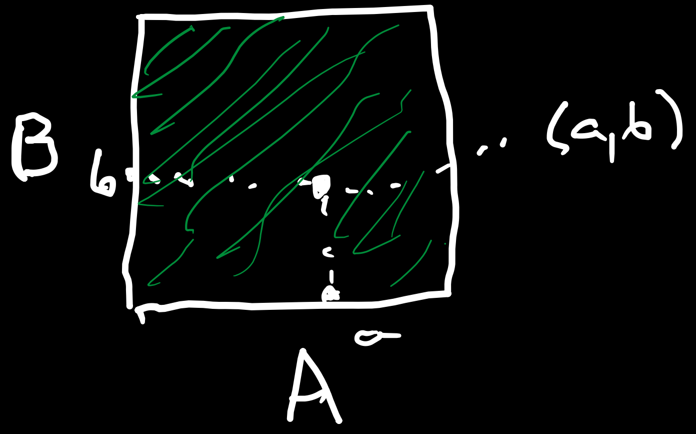
- $\{c\} = \{c,d\}$ niemożliwe, bo $c \neq d$

CZYLI: $\{c,b\} = \{c,d\}$

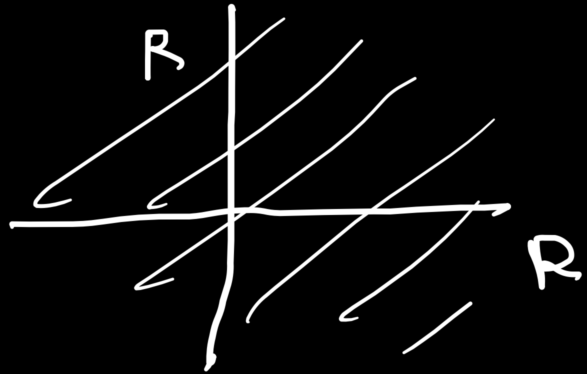
więc: $b = d$.

DEF: (Iloczyn kartezjański zbiorów)

$$A \times B = \{(a,b) : a \in A \wedge b \in B\}$$




P $\quad \mathbb{R} \times \mathbb{R} \ (= \mathbb{R}^2)$




płaszczyzna

P $\quad (\mathbb{R} \times \mathbb{R}) \times \mathbb{R}$ ← przestrz. 3-wymiarowa

$((\mathbb{R} \times \mathbb{R}) \times \mathbb{R}) \times \mathbb{R}$ ← prz. 4-wymiarowa

Fakt. $(A \cup B) \times C = (A \times C) \cup (B \times C)$

D-d. Wystarczy pokaz. że dla dowolnej pary $(x,y)$ mamy $(x,y) \in L \equiv (x,y) \in P$. !!!

$\underline{(x,y) \in (A \cup B) \times C} \equiv (x \in A \cup B) \wedge (y \in C) \equiv$

$(x \in A \vee x \in B) \wedge (y \in C) \equiv$

$(x \in A \wedge y \in C) \vee (x \in B \wedge y \in C) \equiv$

$(x,y) \in A \times C \vee (x,y) \in B \times C \equiv$

$\underline{(x,y) \in (A \times C) \cup (B \times C)}$ $\square$

F. $(A \cap B) \times C = (A \times C) \cap (B \times C)$

P $(A \times B = B \times A) \equiv (A = \emptyset) \vee (B = \emptyset) \vee (A = B)$

F. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

P $(A \times B) \setminus (C \times D) = ((A \setminus C) \times (B \setminus D)) \cup ((A \cap C) \times (B \setminus D)) \cup$

$\cup ((A \setminus C) \times (B \cap D))$ !!!


Zadanie

$A, C \subseteq \Omega_1$

$B, D \subseteq \Omega_2$

F. $X$ nie jest łączny

FAKT. $(\mathbb{R}\times\mathbb{R})\times\mathbb{R} \neq \mathbb{R}\times(\mathbb{R}\times\mathbb{R})$

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Ustalmy zbiór $\Omega$. Ustalmy funkcje zdaniowe

$\varphi, \psi : \Omega \longrightarrow \{0, 1\}$.

$(\varphi\wedge\psi)(\omega) = \varphi(\omega)\wedge\psi(\omega)$

$\omega\in\{x\in\Omega : \varphi(x)\}\cap\{x\in\Omega : \psi(x)\} \equiv$

$\omega\in\{x\in\Omega : \varphi(x)\} \wedge \omega\in\{x\in\Omega : \psi(x)\} \equiv$

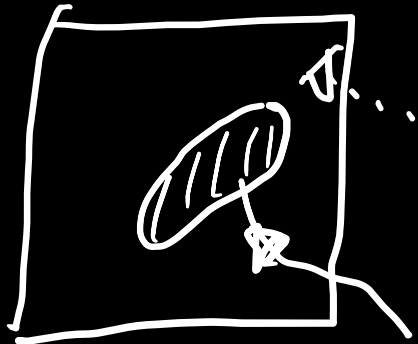
$(\omega\in\Omega \wedge \varphi(\omega)) \wedge (\omega\in\Omega \wedge \psi(\omega)) \equiv$

$\equiv (\omega\in\Omega \wedge \varphi(\omega)\wedge\psi(\omega)) \equiv (\omega\in\Omega \wedge \varphi\wedge\psi(\omega)) \equiv$

$$\equiv \quad \omega \in \{x \in \Omega : (\varphi \wedge \psi)(x)\}.$$

F $\quad \{x \in \Omega : \varphi(x)\} \cap \{x \in \Omega : \psi(x)\} = \{x \in \Omega : (\varphi \wedge \psi)(x)\}$

$\quad$ —— " —— $\cup$ —— " —— $= \{x \in \Omega : (\varphi \vee \psi)(x)\}$



F $\quad \{x \in \Omega : \varphi(x)\}^c = \{x \in \Omega : (\neg\varphi)(x)\}$

$$(\neg\varphi)(x) \overset{\text{def.}}{=} \neg(\varphi(x))$$

$\Omega$

$\{x \in \Omega : \varphi(x)\}$

# KWANTYFIKATORY

Ustalony $\Omega$, $\varphi : \Omega \longrightarrow \{0, 1\}$

**DEF.** $(\forall x)\, \varphi(x) \overset{\text{def}}{\equiv} \{x \in \Omega : \varphi(x)\} = \Omega$

$(\exists x)\, \varphi(x) \equiv \{x \in \Omega : \varphi(x)\} \neq \emptyset$

$\forall$ - kw. uniwersalny

$\exists$ - kw. egzystencjalny

$(\forall x)\, \varphi(x) \equiv \bigwedge_x \varphi(x)$

$(\exists x)\, \varphi(x) \equiv \bigvee_x \varphi(x)$

P $\quad \Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$

$$(\forall x)\, \varphi(x) \equiv \{x \in \Omega : \varphi(x)\} = \Omega$$

$$\equiv \varphi(\omega_1) = 1 \wedge \varphi(\omega_2) = 1 \wedge \dots \wedge \varphi(\omega_n) = 1$$

$$\equiv \bigwedge_{i=1}^{n} (\varphi(\omega_i)) = \varphi(\omega_1) \wedge \varphi(\omega_2) \wedge \dots \wedge \varphi(\omega_n)$$

$$(\exists x)\, \varphi(x) \equiv \varphi(\omega_1) \vee \varphi(\omega_2) \vee \dots \vee \varphi(\omega_n)$$

- $\forall \equiv$ uogólnienie $\wedge$
- $\exists \equiv$ — " — $\vee$