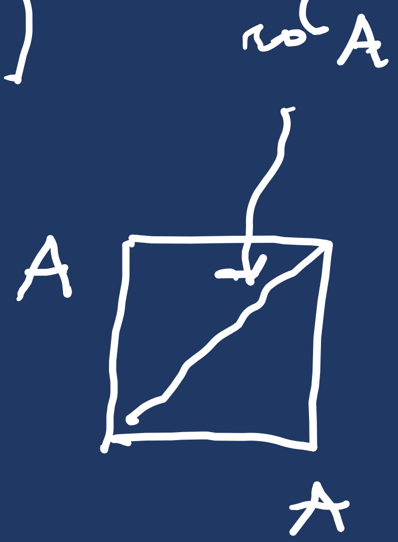
Równoliczność: $A \sim B \equiv (\exists f)\left(f: A \xrightarrow[na]{1-1} B\right)$

$\Updownarrow$

$|A| = |B|$



• $\boxed{A \sim A}$

zwrotność

$id_A = \{(x,x) : x \in A\}$

$id_A : A \xrightarrow[na]{1-1} A$

$id_A = I_A$

symetria

• $\boxed{(A \sim B) \to (B \sim A)}$

$f: A \xrightarrow[na]{1-1} B$

$\Downarrow$

$f^{-1}: B \xrightarrow[na]{1-1} A$

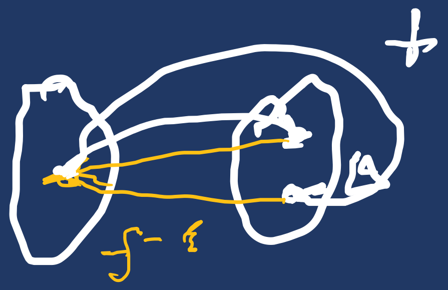
• $\boxed{(A \sim B) \wedge (B \sim C) \to (A \sim C)}$

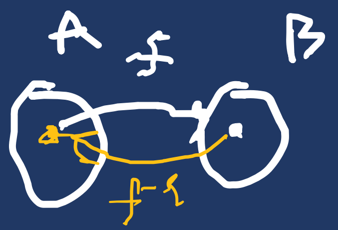
przechodność

$A \xrightarrow[1-1,\ na]{f} B \xrightarrow[1-1\ na]{g} C$

$g \circ f$

$1-1,\ na$





Def
$$\begin{cases} 1) \ |A| = n \equiv A \sim \{1, \dots, n\} & n \geq 1 \\ 2) \ |A| = 0 \equiv A = \emptyset \end{cases}$$

UWAGA: $|A| = \aleph_0 \equiv A \sim \mathbb{N}$, $|A| = \mathfrak{c} \equiv A \sim \mathbb{R}$ } później

$\aleph_0$ = alef zero

$\mathfrak{c}$ = kontinuum

Rodziny indeksowane:

$$F : I \longrightarrow \text{zbiory}$$

1) F jest funkcją
2) $\mathrm{dom}(F) = I$
3) $(\forall i \in I)(F(i) - \text{zbiór})$

$$F = (A_i)_{i \in I} \quad ; \quad A_i = F(i)$$

USTALMY $(A_i)_{i \in I}$

$$x \in \bigcup_{i \in I} A_i \equiv (\exists i \in I)(x \in A_i) \qquad x \in \bigcap_{i \in I} A_i \equiv (\forall i \in I)(x \in A_i)$$

(P) $I = \{1, 2\}$ $\quad A_1, A_2$ – ~~ciąg~~ $\quad (A_i)_{i \in I}$

$$x \in \bigcup_{i \in I} A_i \equiv (\exists i \in \{1,2\})(x \in A_i)$$

$$\equiv x \in A_1 \vee x \in A_2 \equiv x \in A_1 \cup A_2$$

$$\bigcup_{i \in I} A_i = A_1 \cup A_2$$

$$\bigcap_{i \in I} A_i = A_1 \cap A_2$$

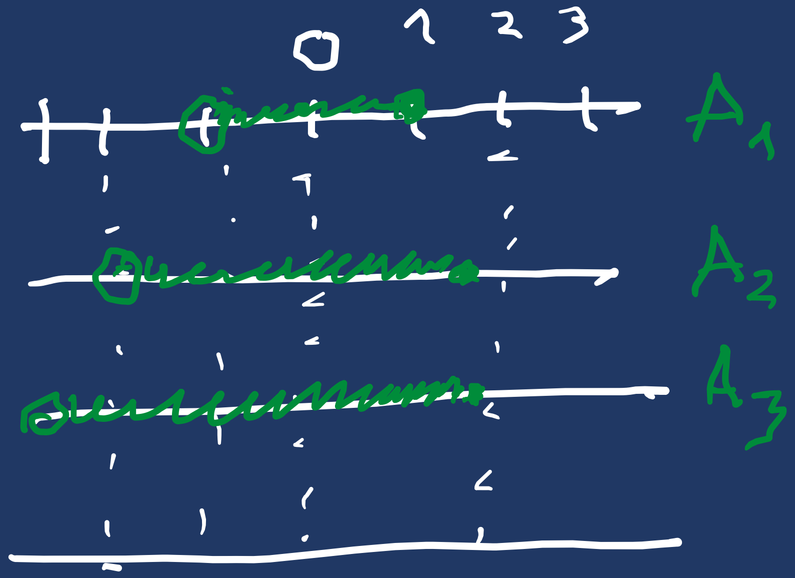
WNIOSEK:

$$\bigcup_{i \in I} A_i = \bigcup \{A_i : i \in I\}$$

$$\bigcap_{i \in I} A_i = \bigcap \{A_i ; i \in I\} \qquad I \neq \emptyset$$

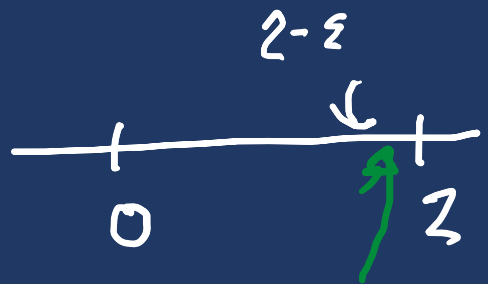
Ⓟ $I = \{1, 2, 3, \dots\} \;\; (= \mathbb{N} \setminus \{0\})$

$$A_i = \left(-i,\, 2 - \frac{1}{i}\right]$$



$$\bigcup_{n \geq 1} A_n = (-\infty, 2)$$

1) $x \geq 0$: $x \in \bigcup_{n \geq 1} A_n \longleftrightarrow 0 \leq x < 2$



2) $x < 0$: $x \in \bigcup_{n \geq 1} A_n \longleftrightarrow x < 0$

$$2 - \varepsilon < 2 - \frac{1}{n} < 2$$

(P) $I = \mathbb{N}^+ \times \mathbb{N}^+$ $\qquad \mathbb{N}^+ = \mathbb{N} \setminus \{0\}$

$F : I \longrightarrow$ zbiory

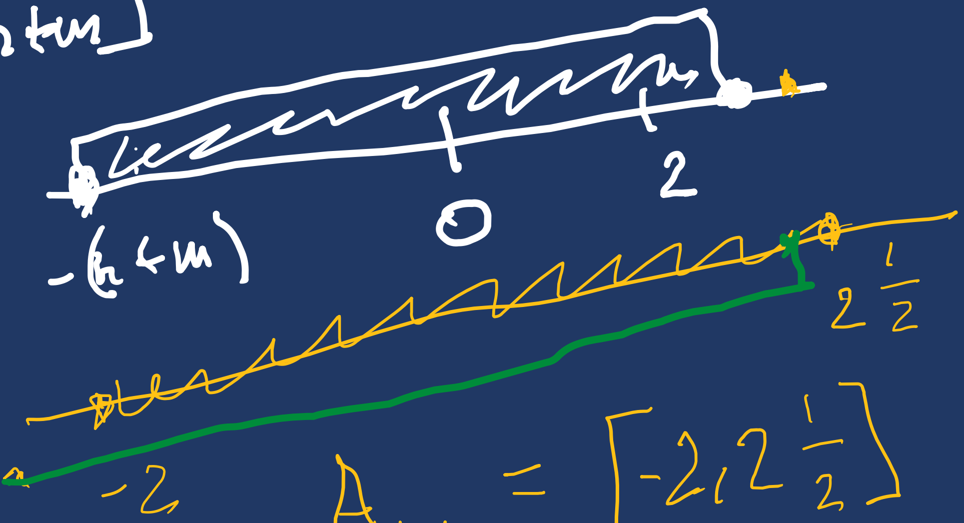
$A_{(n,m)} = F((n,m))$

$A_{n,m} = F((n,m))$

$A_{n,m} = \left[-(n+m),\ 2 + \frac{1}{n+m}\right]$

$\bigcup_{n,m} A_{n,m} = \left(-\infty, 2\frac{1}{2}\right]$

$2 + \frac{1}{2} = 2\frac{1}{2}$

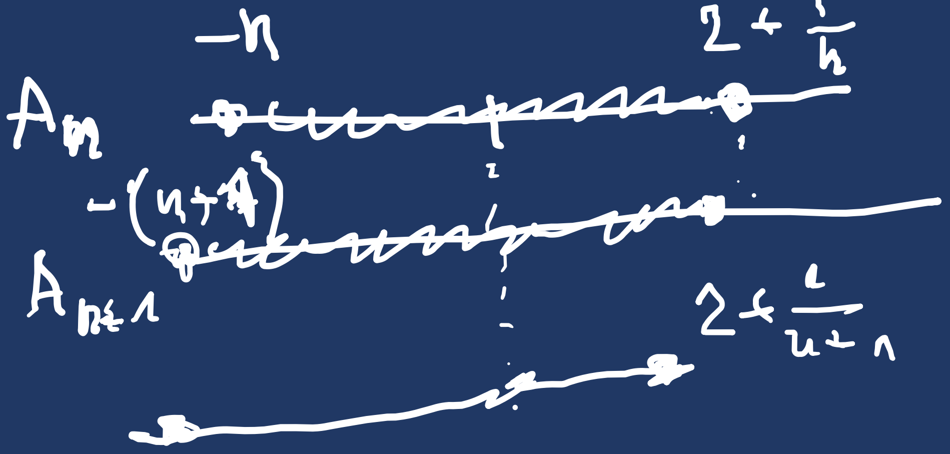


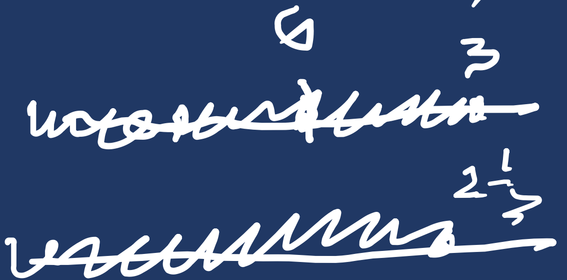
$A_{1,1} = \left[-2, 2\frac{1}{2}\right]$

(P) $(A_n)_{n \geq 1}$ $\qquad A_n = \left[-n, 2+\frac{1}{n}\right]$

$$\bigcap_{n\geq 1} \bigcup_{m \geq n} A_m = \bigcap_{n \geq 1} C_n = \bigcap_{n\geq 1} \left(-\infty, 2+\frac{1}{n}\right] = (-\infty, 2]$$

$$C_n = \bigcup_{m \geq n} A_m = \left(-\infty, 2+\frac{1}{n}\right]$$



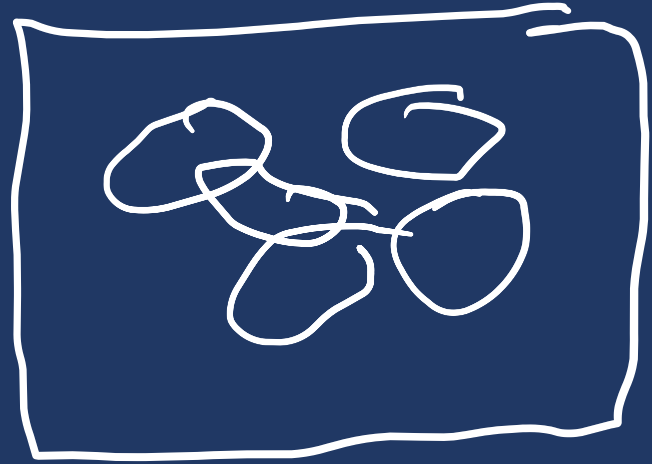


$$\bigcap_{n \geq 0} \bigcup_{m \geq n} A_m = \limsup_{n \to \infty} A_n$$

$$\bigcup_{n \geq 0} \bigcap_{m \geq n} A_m = \liminf_{n \to \infty} A_n$$

$$\boxed{\bigcap_n A_n \subseteq \liminf A_n \subseteq \limsup_n A_n \subseteq \bigcup_n A_n}$$

$$(\forall i \in I)\,(A_i \subseteq \Omega\}$$



Ω

$$\begin{cases} \bigcup_{i \in I} A_i \subseteq \Omega \\ \bigcap_{i \in I} A_i \subseteq \Omega \end{cases}$$

$$x \in \Big(\bigcup_{i \in I} A_i\Big)^c \equiv x \notin \bigcup_{i \in I} A_i \equiv \neg\Big(x \in \bigcup_{i \in I} A_i\Big) \equiv$$

$x \in \Omega$

$$\equiv \neg(\exists i \in I)(x \in A_i] \equiv (\forall i \in I)(x \notin A_i]$$

$$\equiv (\forall i \in I)(x \in A_i^c) \equiv x \in \bigcap_{i \in I} A_i^c .$$

$$\boxed{\begin{aligned} \Big(\bigcup_{i\in I} A_i\Big)^c &= \bigcap_{i\in I} A_i^c \\ \Big(\bigcap_{i\in I} A_i\Big)^c &= \bigcup_{i\in I} A_i^c \end{aligned}}$$

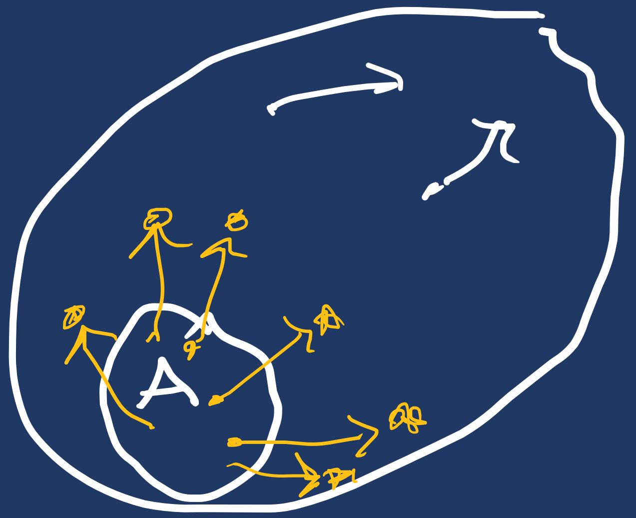
prawa de Morgana dla działań uogóln.

? Q: prawa de Morgana dla $\liminf_n A_n$, $\limsup_n A_n$?

OBRAZY i PRZECIW-OBRAZY

Ustalmy relację R, zbiór A :

$R \subseteq X \times X$



$$R[A] = \{y : (\exists x \in A)((x,y) \in R\}$$

↖ obraz A przez relację R

UWAGA : $f : A \longrightarrow B$

$$\mathrm{rng}(f) = f[\mathrm{dom}(f)]$$

F1. $\boxed{R[A \cup B] = R[A] \cup R[B]}$

$$y \in R[A \cup B] \equiv (\exists x \in A \cup B)((x,y) \in R)$$

$$\equiv (\exists x)(x \in A \cup B \wedge (x,y) \in R) \equiv$$

$$\equiv (\exists x)((x \in A \vee x \in B) \wedge ((x,y) \in R))$$

$$\equiv (\exists x)((x \in A \wedge (x,y) \in R) \vee (x \in B \wedge (x,y) \in R))$$

$$\equiv (\exists x)(x \in A \wedge (x,y) \in R) \vee (\exists x)(x \in B \wedge (x,y) \in R)$$

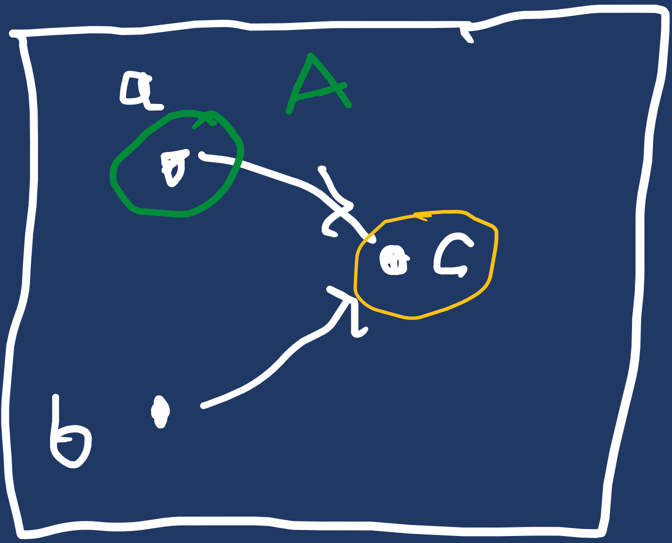
$$\equiv (\exists x \in A)((x,y) \in R) \vee (\exists x \in B)((x,y) \in R) \equiv$$

$$\equiv y \in R[A] \vee y \in R[B] \equiv y \in R[A] \cup R[B]$$

zadanie: $\boxed{R\left[\bigcup_{i \in I} A_i\right] = \bigcup_{i \in I} R[A_i]}$

F2 $\quad \boxed{R[A \cap B] \subseteq R[A] \cap R[B]}$

P



$$(\exists x)\varphi(x) \wedge \psi(x) \to \big((\exists x)\varphi(x) \wedge (\exists x)\psi(x)\big)$$

$R = \{(a,c),(b,c)\}$

$a \neq b$

$A = \{a\}$; $B = \{b\}$

$R[A \cap B] = R[\emptyset] = \emptyset$

$R[A] \cap R[B] = \{c\} \cap \{c\} = \{c\}$

$R^{-1}[A]$ = przeciwobraz A za pomocą R

$$R^{-1}[A] = (R^{-1})[A]$$

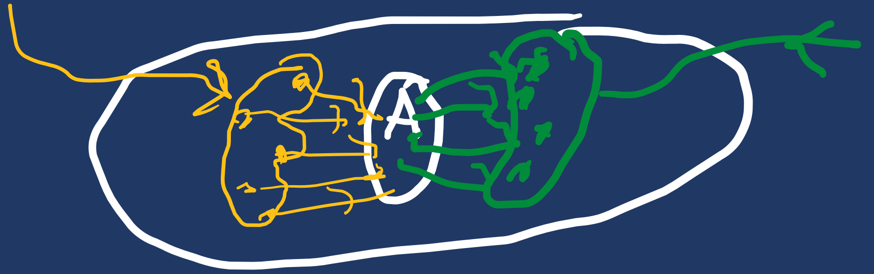
$$x \in \underbrace{R^{-1}}_{Q}[A] \equiv x \in Q[A] \equiv (\exists a \in A)((a,x) \in Q)$$

$$\equiv (\exists a \in A)((a,x) \in R^{-1})$$

$$\equiv (\exists a \in A)((x,a) \in R)$$

$$R^{-1}[A] = \{x : (\exists a \in A)((x,a) \in R)\}$$

$R^{-1}[A]$



$R[A]$