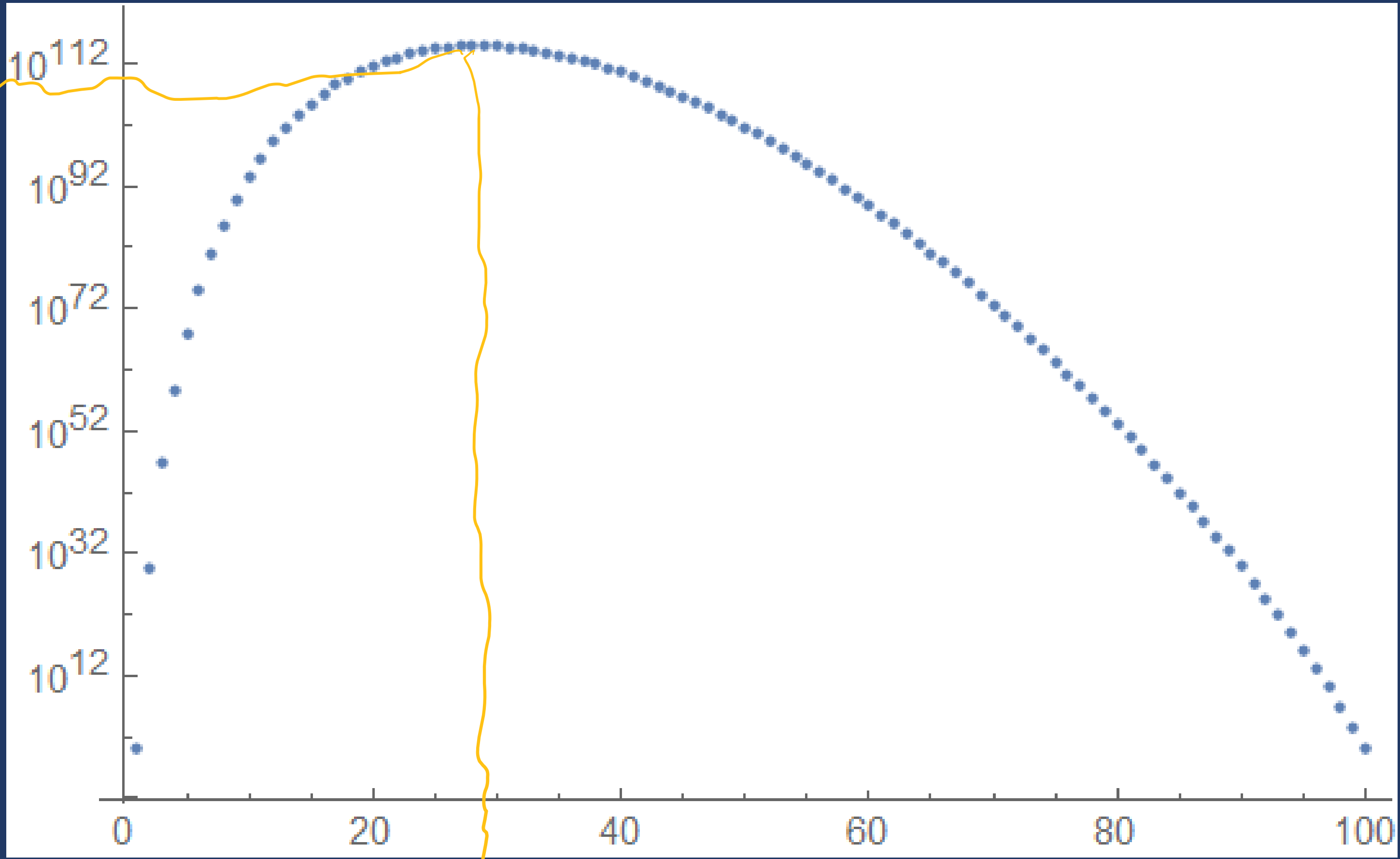
LICZBY STIRLINGA II RODZAJU

$\left(\frac{n}{\ln n}\right)^n \rightarrow$

mało dokładne



$\sim \frac{n}{\ln n}$

$\left(\left\{ {100 \atop k} \right\}\right)_{k=1..100}$

$$\left\{ n \atop k \right\} = \left| \left\{ P : P\text{-part. } [n] \wedge |P| = k \right\} \right|$$

P-part X $\equiv$

1) $\bigcup P = X$

2) $(\forall A)(A \in P \to A \neq \emptyset)$

3) $(\forall A,B)((A,B \in P \wedge A \neq B) \to A \cap B = \emptyset)$.

X

Wniosek: $P \leftarrow \emptyset$:

1) $\bigcup \emptyset = \emptyset$ $\quad \emptyset \downarrow$

2) $(\forall A)(A \in P \to A \neq \emptyset)$ ok

3) ok

$$\left\{ 0 \atop 0 \right\} = 1$$

$$n > 0 \to \left\{ n \atop 0 \right\} = 0$$

$$\left\{ n \atop 0 \right\} = [\![n = 0]\!]$$

Tw. $\left\{ n \atop k \right\} = \frac{1}{k!} \left| Sur([n], [k]) \right|$

$[0] = \emptyset$

$[n] = \{1, \dots, n\}$

Zasada włącz - wył:

ZADANIE: $\left| Sur([n],[k]) \right| = \sum_{a=0}^{k} (-1)^a \binom{k}{a} (k-a)^n$

Wn.

$$\left\{ n \atop k \right\} = \frac{1}{k!} \sum_{a=0}^{k} (-1)^a \binom{k}{a} (k-a)^n$$

(P)

$$\left\{ n \atop 3 \right\} = \frac{1}{3!} \sum_{a=0}^{3} (-1)^a \binom{3}{a} (3-a)^n = \frac{1}{3!} \left(3^n - \binom{3}{1} 2^n + \dots \right) = \dots$$

$$= \frac{1}{3!} \left(3^n - 3 \cdot 2^n + 3 - [\![n = 0]\!] \right)$$

$$\left\{ {n \atop 3} \right\} = \frac{1}{3!}\left(3^n - 3\cdot 2^n + 3 - [[n=0]]\right) =$$

$$= \frac{1}{6}3^n\left(1 - 3\underbrace{\left(\frac{2}{3}\right)^n}_{\downarrow\, 0} + 3\cdot\underbrace{\left(\frac{1}{3}\right)^n}_{\downarrow\, 0} - \underbrace{\frac{[[n=0]]}{3^n}}_{\downarrow\, 0}\right) \sim \frac{3^n}{6}.$$

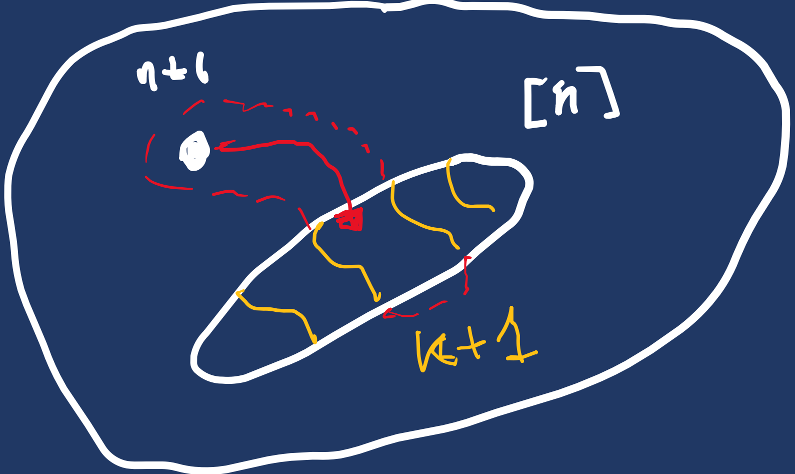
$$\left\{ {n \atop 3} \right\} \sim \frac{1}{3!}3^n$$

Rekursja $\left\{ {n+1 \atop k+1} \right\} = \left\{ {n \atop k} \right\} + \left\{ {n \atop k+1} \right\} (k+1)$

$$\left| \{ P : P\text{-par. } [n+1] \wedge |P| = k+1 \wedge \{n+1\} \in P \} \right| = \left\{ {n \atop k} \right\}$$

Zadanie

zaprogramuj to



$$\boxed{\left\{ {n+1 \atop k+1} \right\} = (k+1) \left\{ {n \atop k+1} \right\} + \left\{ {n \atop k} \right\}}$$

$$\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}$$

$$\left\{ {n \atop n} \right\} = 1 \; ; \; \left\{ {n \atop 0} \right\} = [n=0]$$

Ustalmy $x \in \mathbb{N}_{>0}$; $n \in \mathbb{N}$

$f : [n] \longrightarrow [x]$

- $\mathrm{rng}(f) \subseteq [x]$
- $f : [n] \xrightarrow[\text{surj.}]{} \mathrm{rng}(f)$

$$x^n = \left|[x]^{[n]}\right| = $$

$$= \left| \bigcup_{A \subseteq [x]} \mathrm{Sur}([n], A) \right| =$$

$$= \sum_{A \subseteq [x]} \left| \mathrm{Sur}([n], A) \right| = \sum_{A \subseteq [x]} |A|! \left\{ {n \atop |A|} \right\} =$$

$$= \sum_{k=0}^{x} \sum_{\substack{A \subseteq [x] \\ |A| = k}} |A|! \left\{ {n \atop |A|} \right\} = \sum_{k=0} \sum_{\substack{A \subseteq [x] \\ |A| = k}} k! \left\{ {n \atop k} \right\} = \sum_{k=0}^{x} \binom{x}{k} \left\{ {n \atop k} \right\} k! =$$

$$\sum_{k=0}^{x} \binom{x}{k} \left\{ n \atop k \right\} k! = \sum_{k=0}^{x} \frac{x^{\underline{k}}}{\cancel{k!}} \left\{ n \atop k \right\} \cancel{k!} =$$

$$= \sum_{k=0}^{x} x^{\underline{k}} \left\{ n \atop k \right\} = \sum_{k=0}^{n} \left\{ n \atop k \right\} x^{\underline{k}} \qquad x \geq n$$

$\forall x \in \mathbb{C}$

$$x^n = \sum_{k=0}^{n} \left\{ n \atop k \right\} x^{\underline{k}} \longleftarrow x \in \mathbb{N},\ x \geq n$$

wiel. st n

$$(x^m)' = m x^{m-1}$$

$$(x^m)'' = m(m-1) x^{m-2}$$

$$\cancel{\left\{ {n \atop k} \right\}} \qquad \boxed{\left\{ {n+1 \atop k} \right\} = \sum_{a=0}^{n} \binom{n}{a} \left\{ {n-a \atop k-1} \right\}}$$

$$\begin{array}{cccc}
\left\{ {0 \atop 0} \right\} & & & \\
\left\{ {1 \atop 0} \right\} & \left\{ {1 \atop 1} \right\} & & \\
\left\{ {2 \atop 0} \right\} & \left\{ {2 \atop 1} \right\} & \left\{ {2 \atop 2} \right\} & \\
\left\{ {3 \atop 0} \right\} & \left\{ {3 \atop 1} \right\} & \left\{ {3 \atop 2} \right\} & \left\{ {3 \atop 3} \right\}
\end{array}$$

$$\left\{ {3 \atop 2} \right\} = \sum_{a=0}^{2} \binom{2}{a} \left\{ {2-a \atop 1} \right\}$$

DEF: Liczby Bella

$$B_n = \sum_k \left\{ {n \atop k} \right\}$$

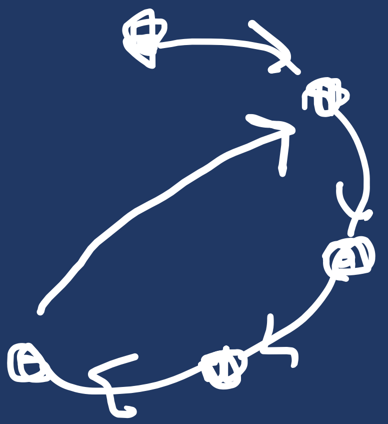
$$B_n = \left| \{ P : P \text{ - part. } [n] \} \right|$$

Wn. $$B_{n+1} = \sum_{a=0}^{n} \binom{n}{a} \cdot B_{n-a}$$

Zadanie

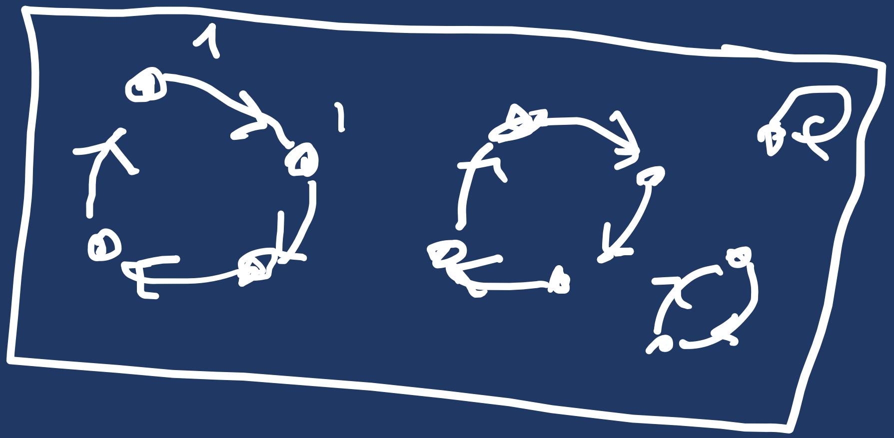
LICZBY STIRLINGA I RODZAJU.

- $\pi \in Sym(X)$
  $X = [n]$



niemożliwe

generuje rozbicie X
na rozł. cykle



X

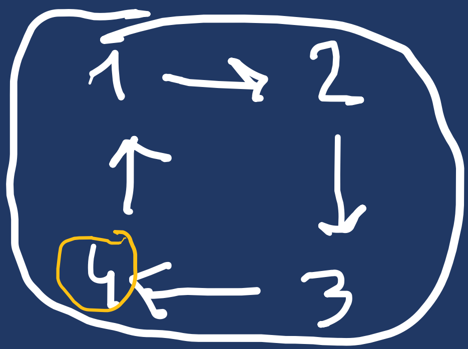
$$\left\langle {n \atop k} \right\rangle = \left| \left\{ \pi \in \mathrm{Sym}([n]) : \pi \text{ rozkłada się na } k \text{ cykli} \right\} \right|$$

$$\left\langle {n \atop n} \right\rangle = 1$$

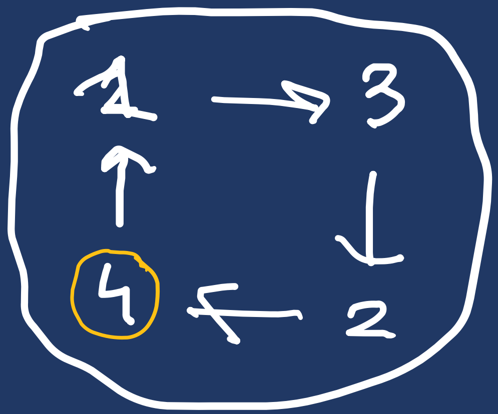
$$\left\langle {n \atop n-1} \right\rangle = \binom{n}{2}$$

$$\left\langle {n \atop 1} \right\rangle = ?$$

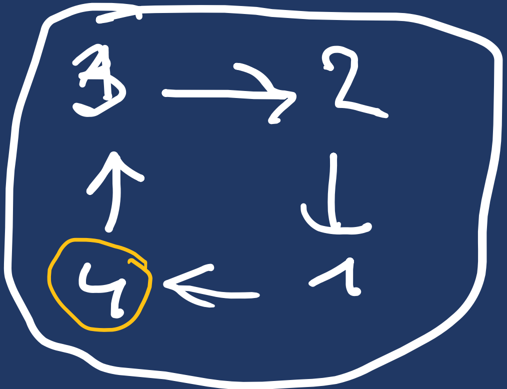
$n=4$



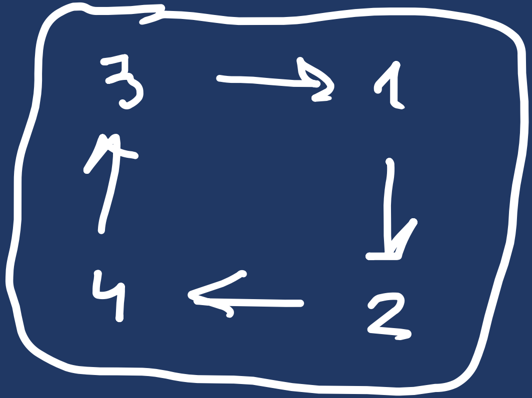
$4 : 1\,2\,3$



$4 : 1\,3\,2$

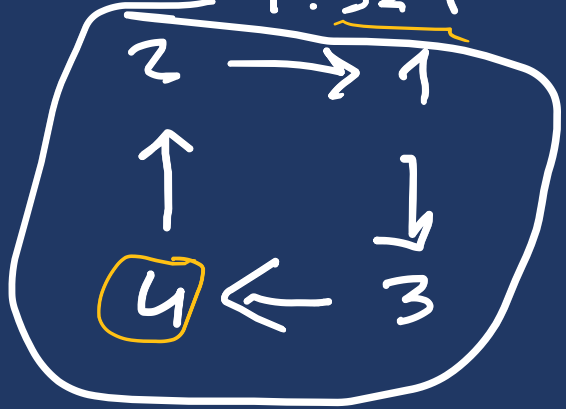


$4 : 3\,2\,1$

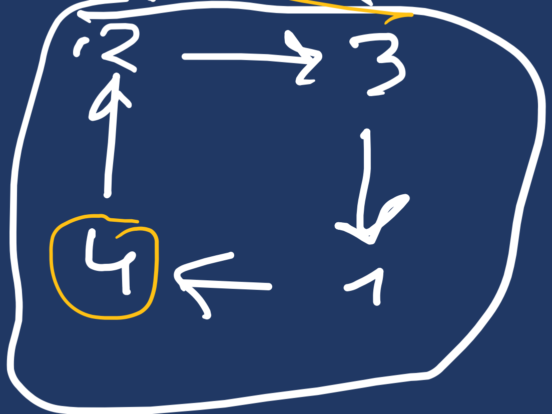


$4 : 3\,1\,2$

$$\boxed{\left\langle {n \atop 1} \right\rangle = (n-1)!}$$



$4 : 2\,1\,3$



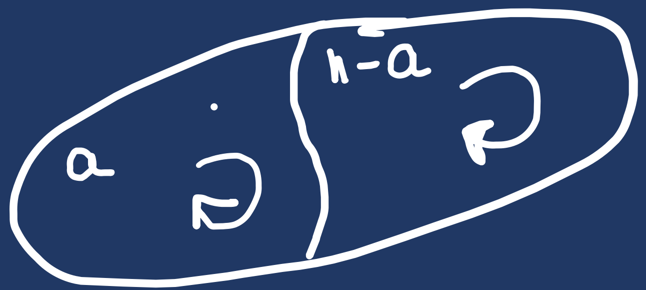
$4 : 2\,3\,1$

$$\sum_{k=0}^{n} \left\langle {n \atop k} \right\rangle = n!$$

$$\left\langle {n \atop 1} \right\rangle = (n-1)!$$

$$\left\langle {n \atop 2} \right\rangle = \text{?}$$

$$\left\langle {n \atop 2} \right\rangle = \frac{1}{2} \sum_{a=1}^{n-1} \binom{n}{a} (a-1)!\,(n-a-1)!$$



n

$1 \le a \le n-1$

$$= \frac{1}{2} \sum_{a=1}^{n-1} \frac{n!}{a!\,(n-a)!} (a-1)!\,(n-a-1)!$$

$$= \frac{n!}{2} \sum_{a=1}^{n-1} \frac{1}{a(n-a)} =$$

$$= \frac{n!}{2} \sum_{a=1}^{n-1} \frac{1}{a(n-a)} = \frac{n!}{2} \cdot \frac{1}{n} \sum_{a=1}^{n-1} \left(\frac{1}{a} + \frac{1}{n-a} \right) =$$

$$\frac{1}{a(n-a)} = \frac{1}{n} \left(\frac{1}{a} + \frac{1}{n-a} \right)$$

$$= \frac{(n-1)!}{2} \left(\sum_{a=1}^{n-1} \frac{1}{a} + \sum_{a=1}^{n-1} \frac{1}{n-a} \right) =$$

$b = n-a$

$$= \frac{(n-1)!}{2} \cdot 2 \cdot H_{n-1} = \boxed{\left\langle {n \atop 2} \right\rangle = (n-1)!\, H_{n-1}}$$

$$\left\langle {n \atop 2} \right\rangle = (n-1)!\, H_{n-1}$$

WNIOSEK

$$\frac{\left\langle {n \atop 2} \right\rangle}{n!} = \frac{(n-1)!\, H_{n-1}}{n!} = \frac{H_{n-1}}{n} \approx \frac{\ln n}{n}.$$

$$\frac{\left\langle {n \atop 1} \right\rangle}{n!} = \frac{(n-1)!}{n!} = \frac{1}{n}$$

$n = 100$:

$$\frac{\left\langle {100 \atop 1} \right\rangle}{100!} \approx \frac{1}{100}$$

$$\frac{\left\langle {100 \atop 2} \right\rangle}{100!} \approx \frac{4}{100}.$$

$$\left\langle {n+1 \atop k+1} \right\rangle = n \left\langle {n \atop k+1} \right\rangle + \left\langle {n \atop k} \right\rangle$$

$$\left\{ {n+1 \atop k+1} \right\} = (k+1) \left\{ {n \atop k+1} \right\} + \left\{ {n \atop k} \right\}$$

$$\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}$$