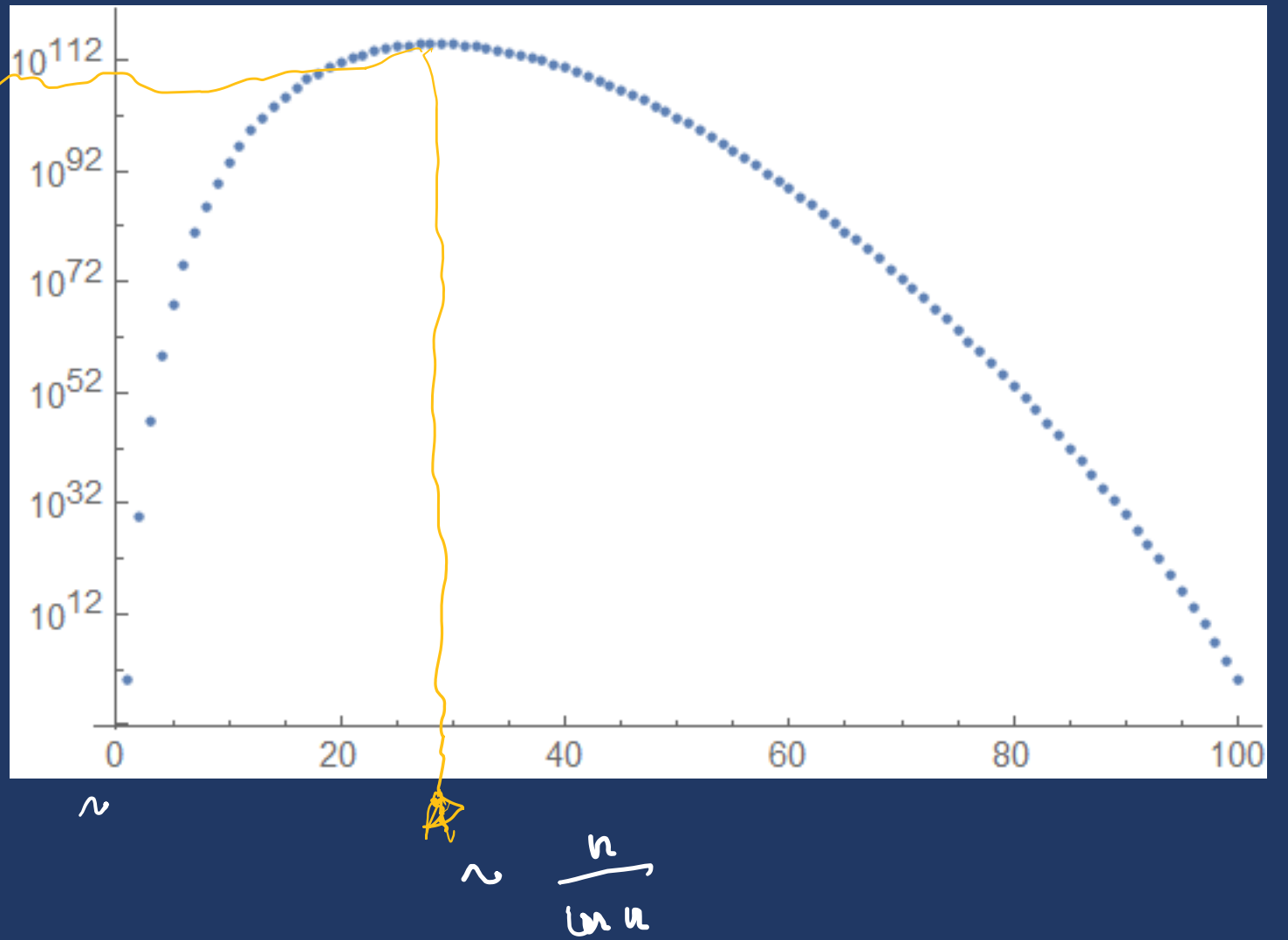


LICZBY STIRLINGA II RODZAJU

$\left(\frac{n}{\ln n}\right)^n \rightarrow$
 mało
 dokładne

$\left\{ \begin{matrix} 100 \\ k \end{matrix} \right\}$
 $k=1-100$



$$\binom{n}{k} = |\{P : P\text{-part. } [n] \wedge |P| = k\}|$$

$$X \text{ P-part } X \equiv 1) \cup P = X$$



$$2) (\forall A)(A \in P \rightarrow A \neq \emptyset)$$

$$3) (\forall A, B)((A, B \in P \wedge A \neq B) \rightarrow A \cap B = \emptyset).$$

$$\text{Wurset: } P \leftarrow \emptyset : 1) \cup \emptyset = \emptyset \quad \downarrow \emptyset$$

$$2) (\forall A)(A \in P \rightarrow A \neq \emptyset) \quad \text{OK}$$

$$3) \text{ OK}$$

$$\binom{0}{0} = 1$$

$$n > 0 \rightarrow \binom{n}{0} = 0$$

$$\binom{n}{0} = \mathbb{I}n=0\mathbb{I}$$

$$\text{Tw. } \left\{ \begin{matrix} n \\ k \end{matrix} \right\} = \frac{1}{k!} | \text{Sur}([n], [k]) | \quad [0] = \emptyset$$

$$[n] = \{1, \dots, n\}$$

Zasada włączenia - wyłączenia:

$$\text{ZADANIE: } | \text{Sur}([n], [k]) | = \sum_{a=0}^k (-1)^a \binom{k}{a} (k-a)^n$$

tw.

$$\left\{ \begin{matrix} n \\ k \end{matrix} \right\} = \frac{1}{k!} \sum_{a=0}^k (-1)^a \binom{k}{a} (k-a)^n$$

(P)

$$\left\{ \begin{matrix} n \\ 3 \end{matrix} \right\} = \frac{1}{3!} \sum_{a=0}^3 (-1)^a \binom{3}{a} (3-a)^n = \frac{1}{3!} (3^n - \binom{3}{1} 2^n + \dots) = \dots$$

$$= \frac{1}{3!} (3^n - 3 \cdot 2^n + 3 \cdot 1^n - [n=0])$$

$$\begin{aligned} \left\{ \begin{matrix} n \\ 3 \end{matrix} \right\} &= \frac{1}{3!} (3^n - 3 \cdot 2^n + 3 - \|\mu \neq 0\|) = \\ &= \frac{1}{6} 3^n \left(1 - 3 \underbrace{\left(\frac{2}{3}\right)^n}_{\downarrow 0} + 3 \cdot \underbrace{\left(\frac{1}{3}\right)^n}_{\downarrow 0} - \underbrace{\frac{\|\mu \neq 0\|}{3^n}}_{\downarrow 0} \right) \sim \frac{3^n}{6}. \end{aligned}$$

$$\left\{ \begin{matrix} n \\ 3 \end{matrix} \right\} \sim \frac{1}{3!} 3^n$$

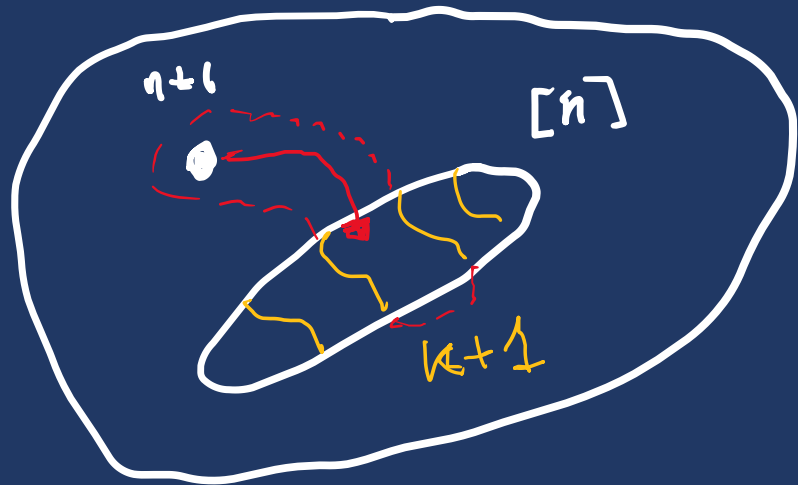
Rekursija

$$\begin{Bmatrix} n+1 \\ k+1 \end{Bmatrix} = \begin{Bmatrix} n \\ k \end{Bmatrix} + \begin{Bmatrix} n \\ k+1 \end{Bmatrix} \quad (k+1)$$



Academy
of programming
to

$$|\{P: P\text{-var. } [n+1] \wedge |P|=k+1 \wedge \{n+1\} \in P\}| = \begin{Bmatrix} n \\ k \end{Bmatrix}$$



$$\begin{Bmatrix} n+1 \\ k+1 \end{Bmatrix} = (k+1) \begin{Bmatrix} n \\ k+1 \end{Bmatrix} + \begin{Bmatrix} n \\ k \end{Bmatrix}$$

$$\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}$$

$$\begin{Bmatrix} n \\ n \end{Bmatrix} = 1 ; \quad \begin{Bmatrix} n \\ 0 \end{Bmatrix} = [n=0]$$

ustawmy $x \in \mathbb{N} > 0; n \in \mathbb{N}$

$$f: [n] \rightarrow [x]$$

$$x^n = |[x]^{[n]}| =$$

$$\bullet \text{rng}(f) \subseteq [x]$$

$$\bullet f: [n] \xrightarrow{\text{surj.}} \text{rng}(f)$$

$$= \left| \bigcup_{A \subseteq [x]} \text{sur}([n], A) \right| =$$

$$= \sum_{A \subseteq [x]} |\text{sur}([n], A)| = \sum_{A \subseteq [x]} |A|! \cdot \left\{ \begin{matrix} n \\ |A| \end{matrix} \right\} =$$

$$= \sum_{k=0}^x \sum_{\substack{A \subseteq [x] \\ |A|=k}} |A|! \cdot \left\{ \begin{matrix} n \\ |A| \end{matrix} \right\} = \sum_{k=0}^x \sum_{\substack{A \subseteq [x] \\ |A|=k}} k! \cdot \left\{ \begin{matrix} n \\ k \end{matrix} \right\} = \sum_{k=0}^x \binom{x}{k} \left\{ \begin{matrix} n \\ k \end{matrix} \right\} k! =$$

$$\sum_{k=0}^x \binom{x}{k} \frac{n!}{k!} = \sum_{k=0}^x \frac{x^{\underline{k}}}{k!} \frac{n!}{k!} =$$

$$= \sum_{k=0}^x x^{\underline{k}} \frac{n!}{k!} = \sum_{k=0}^n \frac{n!}{k!} x^{\underline{k}} \quad x \geq n$$

$\forall x \in \mathbb{N}$

$$x^n = \sum_{k=0}^n \frac{n!}{k!} x^{\underline{k}}$$

wied. st n

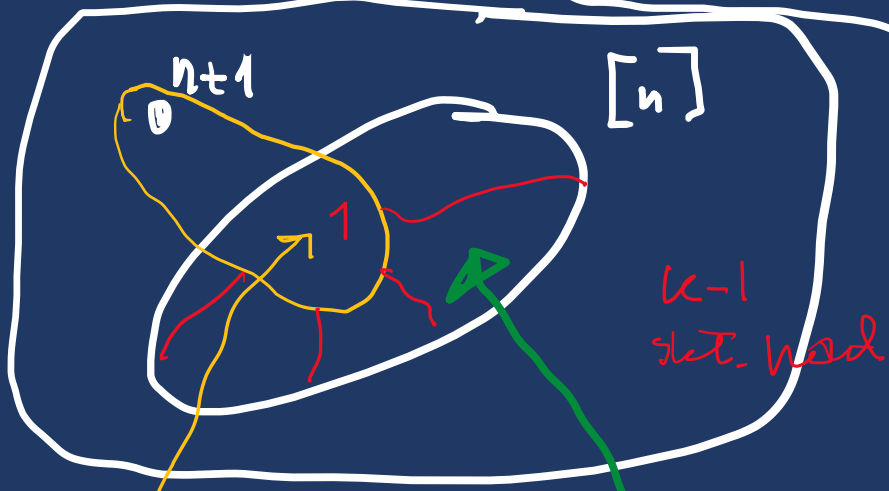
$x \in \mathbb{N}, x \geq n$

$$(x^m)' = m x^{m-1}$$

$$(x^m)' = m(m-1)x^{m-2}$$

~~$$\left\{ \begin{matrix} n \\ k \end{matrix} \right\}$$~~

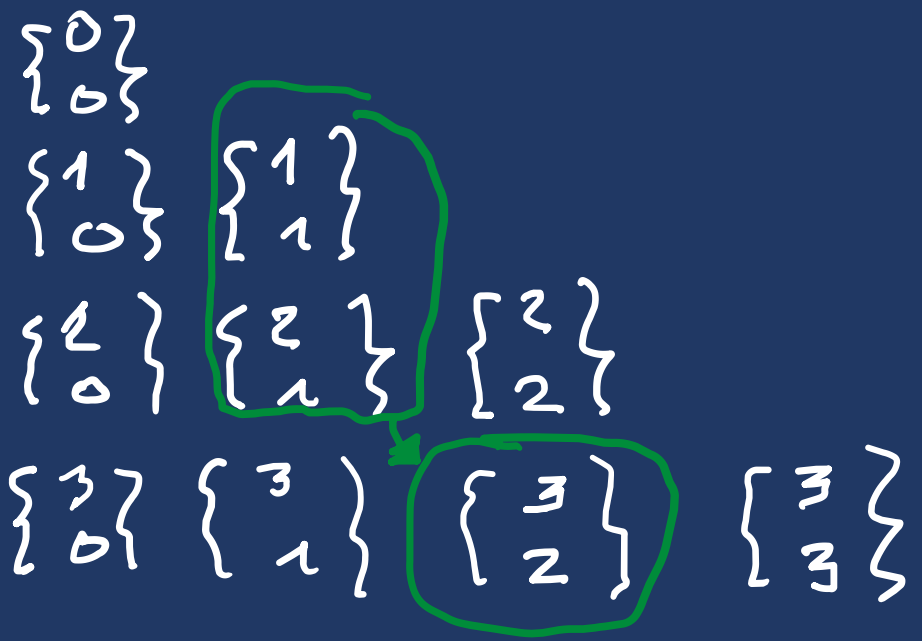
$$\left\{ \begin{matrix} n+1 \\ k \end{matrix} \right\} = \sum_{a=0}^n \binom{n}{a} \left\{ \begin{matrix} n-a \\ k-1 \end{matrix} \right\}$$



a - elem.

n-a elem.

k-1
set. used



$$\left\{ \begin{matrix} 3 \\ 2 \end{matrix} \right\} = \sum_{a=0}^2 \binom{2}{a} \left\{ \begin{matrix} 2-a \\ 1 \end{matrix} \right\}$$

DEF: Liczby Bella

$$B_n = \sum_k \{ \binom{n}{k} \}$$

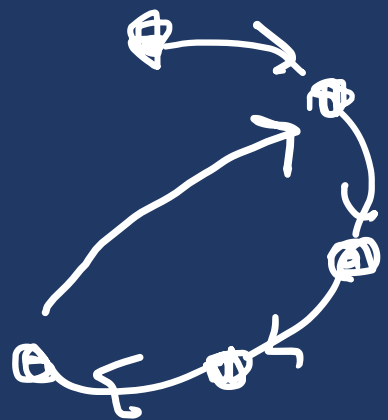
$$B_n = | \{ P: P\text{-part. } [n] \} |$$

Wz. $B_{n+1} = \sum_{a=0}^n \binom{n}{a} \cdot B_{n-a}$

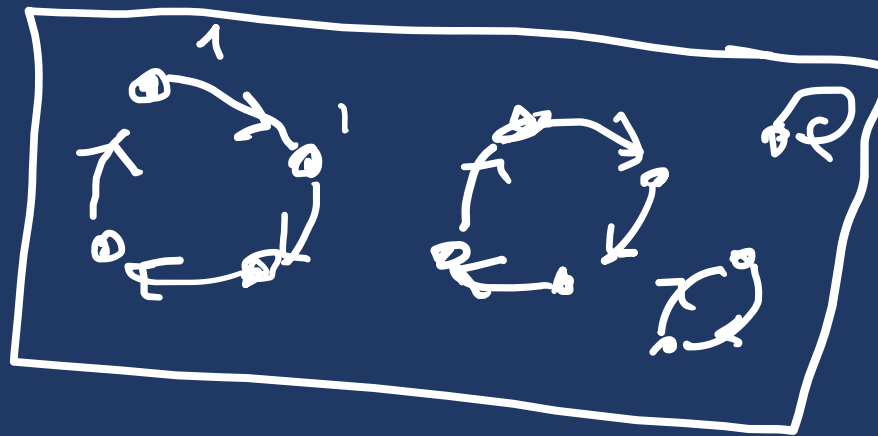
inductive 

LICZBY STIRLINGA I PODZAJU.

- $\pi \in \text{Sym}(X)$
 $X = [n]$



niemożliwe



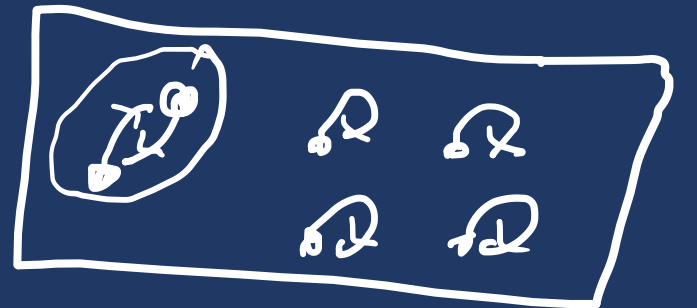
X

generuje rozbięcie X
na rozł. cykle

$$\left\langle \begin{matrix} n \\ k \end{matrix} \right\rangle = \left| \left\{ \pi \in \text{Sym}([n]) : \pi \text{ rozkłada się na } k \text{ cykli} \right\} \right|$$

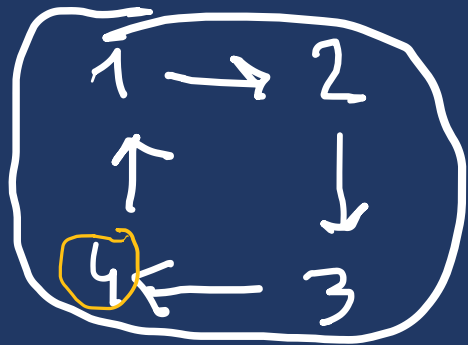
$$\left\langle \begin{matrix} n \\ n \end{matrix} \right\rangle = 1$$

$$\left\langle \begin{matrix} n \\ n-1 \end{matrix} \right\rangle = \binom{n}{2}$$

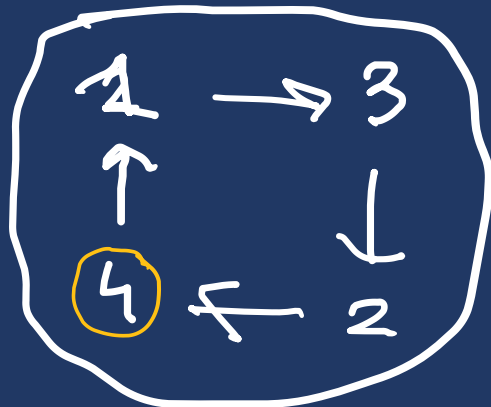


$$\langle \begin{matrix} n \\ 1 \end{matrix} \rangle = 2$$

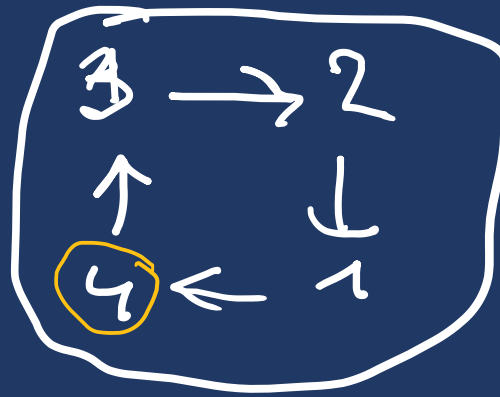
$$n=4$$



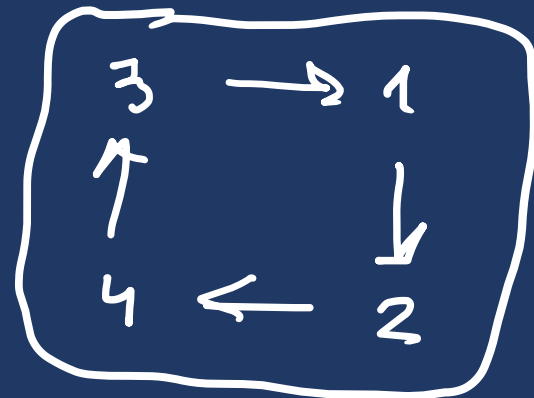
4: 1 2 3



4: 1 3 2

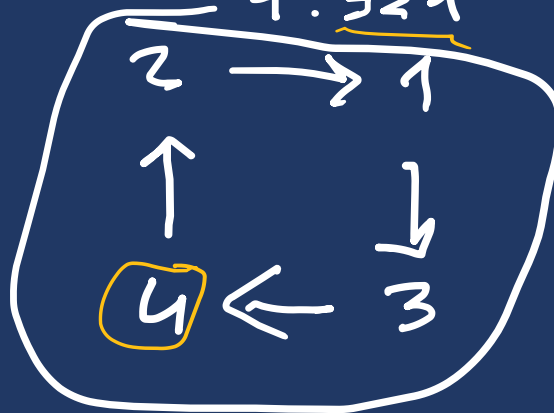


4: 3 2 1

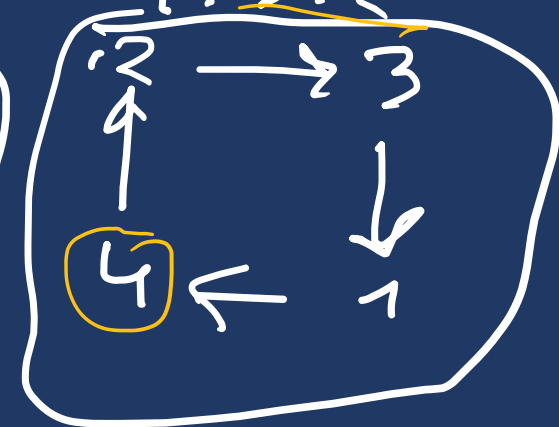


4: 3 1 2

$$\langle \begin{matrix} n \\ 1 \end{matrix} \rangle = (n-1)!$$



4: 2 1 3



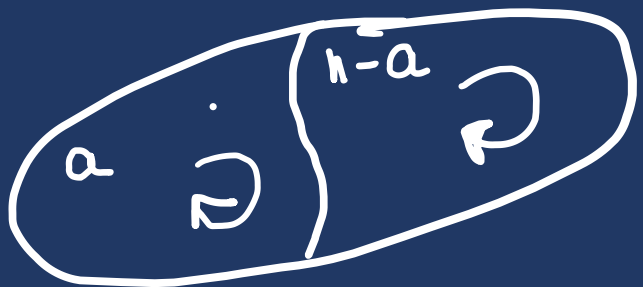
4: 2 3 1

$$\sum_{k=0}^n \binom{n}{k} = 2^n$$

$$\binom{n}{1} = (n-1)!$$

$$\binom{n}{2} = ?$$

$$\binom{n}{2} = \frac{1}{2} \sum_{a=1}^{n-1} \binom{n}{a} (a-1)! (n-a-1)!$$



$$1 \leq a \leq n-1$$

$$= \frac{1}{2} \sum_{a=1}^{n-1} \frac{n!}{a! (n-a)!} (a-1)! (n-a-1)!$$

$$= \frac{n!}{2} \sum_{a=1}^{n-1} \frac{1}{a(n-a)} =$$

$$= \frac{n!}{2} \sum_{a=1}^{n-1} \frac{1}{a(n-a)} = \frac{n!}{2} \frac{1}{n} \sum_{a=1}^{n-1} \left(\frac{1}{a} + \frac{1}{n-a} \right) =$$

$$\frac{1}{a(n-a)} = \frac{1}{n} \left(\frac{1}{a} + \frac{1}{n-a} \right)$$

$$= \frac{(n-1)!}{2} \left(\sum_{a=1}^{n-1} \frac{1}{a} + \sum_{a=1}^{n-1} \frac{1}{n-a} \right) =$$

$$= \frac{(n-1)!}{2} \cdot 2 \cdot H_{n-1}$$

$$\binom{n}{2} = (n-1)! \cdot H_{n-1}$$

$$\binom{n}{2} = (n-1)! H_{n-1}$$

WMOSEK

$$\frac{\binom{n}{2}}{n!} = \frac{(n-1)! H_{n-1}}{n!} = \frac{H_{n-1}}{n}$$

$$\approx \frac{\ln n}{n}$$

$n=100$;

$$\frac{\binom{100}{1}}{100!} \approx \frac{1}{100}$$

$$\frac{\binom{n}{1}}{n!} = \frac{(n-1)!}{n!} = \frac{1}{n}$$

$$\frac{\binom{100}{2}}{100!} \approx \frac{4}{100}$$

$$\langle \begin{matrix} n+1 \\ k+1 \end{matrix} \rangle = n \langle \begin{matrix} n \\ k+1 \end{matrix} \rangle + \langle \begin{matrix} n \\ k \end{matrix} \rangle$$

$$\left\{ \begin{matrix} n+1 \\ k+1 \end{matrix} \right\} = (k+1) \left\{ \begin{matrix} n \\ k+1 \end{matrix} \right\} + \left\{ \begin{matrix} n \\ k \end{matrix} \right\}$$

$$\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}$$