Algorithmic Number Theory - List 3

Task 1 (30 pts)
Calculate the order of random elements of a prime field. To do so:

(a) generate a random prime number p_{10} having at least 100 bits in length,

(b) generate random exponents $\alpha_i \geq 15$, $i = 1, 2, \ldots, 9$, such that the number

$$p = 2^{\alpha_1} \cdot 3^{\alpha_2} \cdot 5^{\alpha_3} \cdot 7^{\alpha_4} \cdot 11^{\alpha_5} \cdot 13^{\alpha_6} \cdot 17^{\alpha_7} \cdot 19^{\alpha_8} \cdot 23^{\alpha_9} \cdot p_{10} + 1$$

is prime,

(c) generate ten random numbers $g_j \in \{2, 3, \ldots, p-2\}$, $j = 1, 2, \ldots, 10$,

(d) for each $j = 1, 2, \ldots, 10$ calculate ordg_j in $\mathbb{F}_p^*$,

(e) make sure that the results ordg_j are correct by implementing tests.