

$$p = 2\tilde{p} + 1$$

do {

generate random prime \tilde{p}

$$n := 2\tilde{p} + 1$$

} while (n is not a prime)

$$p := n;$$

DLP

$$\mathbb{F}_p^* = \{1, 2, \dots, p-1, \tilde{p}\}$$

take g which is random in $\mathbb{F}_p^* \setminus \{1\}$

$$\tilde{g} := g^2 \pmod{p}$$

if $\tilde{g} \neq 1 \pmod{p}$ then $\text{ord } \tilde{g} = \tilde{p}$

take random $x \in \{2, \dots, \tilde{p}-1\}$

$$y = \tilde{g}^x \pmod{p}$$

$$y \in \langle \tilde{g} \rangle$$

$$\text{Pollard-ρ}(\tilde{g}, y, \tilde{p}, p) \rightarrow x'$$

$$x' = x \pmod{\tilde{p}}$$

$$\begin{aligned} |\mathbb{F}_p^*| &= p-1 = \\ &= 2 \cdot \tilde{p} \end{aligned}$$