

add (P, Q)   
 zero at infinity   
 both are in projective coordinates

- if  $P \neq Q$    
 return Q
- if  $Q = \mathcal{O}$    
 return P

$S_1 = \dots$   
 $S_2 = \dots$   
 $u_1 = \dots$   
 $u_2 = \dots$

• if  $((S_1 == S_2) \ \&\& \ (u_1 == u_2))$    
 return double (P)

$w = \dots$   
 $P = \dots$   
 $R = \dots$   
 $Z_3 = \dots$   
 $X_3 = \dots$   
 $Y_3 = [\dots]$

← at first calculate the content of the brackets only

• if ( $Y_3$  is odd)   
 $Y_3 := Y_3 + p$ ;   
 the prime number determining the field.

• shift  $Y_3$  to the right by one bit

$$Y_3 := Y_3 / 2$$

• return  $(X_3, Y_3, Z_3)$

double (P)

- if  $(P == \mathcal{O})$    
 return P.   
 zero at infinity  $(0, 1, 0)$
- if  $(P.Y == 0)$    
 return  $\mathcal{O}$ .   
 the Y coordinate of point P

$$S = 2 \cdot P.Y \cdot P.Z \pmod{p}$$

$B = \dots$   
 $w = \dots$   
 $h = \dots$   
 $X_3 = \dots$   
 $Y_3 = \dots$   
 $Z_3 = \dots$

• return  $(X_3, Y_3, Z_3)$

tests:

$$2P + 3P = 2(2P) + P$$